ISSN: 2277-9655
Impact Factor: 1.852

IJESRT

INTERNATIONAL JOURNAL OF ENGINEERING SCIENCES & RESEARCH TECHNOLOGY

Flow of a Jeffrey Fluid between Finite Deformable Porous Layers

S.Sreenadh[*1], A.Parandhama[2], E.Sudhakara[3], M. Krishna Murthy[4]
[*1,3,4] Department of Mathematics, Sri Venkateswara University, Tirupati- 517502, India
[2] Department of Mathematics, Sree Vidyanikethan Engineering College, Tirupati, India
profsreenadh@gmail.com

Abstract

The flow of a Jeffrey fluid between a thin deformable porous layers is investigated. The governing equations are solved in the free flow and porous flow regions. The expressions for the velocity field and deformation are obtained. When $\lambda_1 \to 0$, the results agree with the corresponds ones of Barry et al (1991). The effects of Jeffrey parameter, the pressure gradient, ϕ^f and ϕ^s on the flow velocity and deformation are discussed. It is found that the velocity increases with the increase in the non-Newtonian Jeffrey parameter λ_1. This study is also relevant to filtration technology, soil mechanics and to other biological problems such as the mechanics of articular cartilage.

Keywords: Jeffrey Fluid, Finite Deformable.

Introduction

Viscous flow through and past porous media has important applications in engineering and medicine. Most of the research works available deal with flow through rigid porous media. But when a biofluid flows in a physiological system, there will be an interaction between free flow and tissue (deformable) regions. Thus the study on free flow in a deformable porous layer is necessitated. Further most of biological fluids are observed to be non-Newtonian and these fluids may be modelled with a simple elegant Jeffrey model.

The study of deformation in porous materials with coupled fluid movement was initiated by Terzaghi [1] and later continued by Biot [2,3,4] into a successful theory of soil consolidation and acoustic propagation. Atkin and Craine [5], Bowen [6] and Bedford and Drumheller [7] made important works on the theory of mixtures. Mow et al. [8] developed a similar theory for the study of biological tissue mechanics. Using this theory arterial wall permeability is discussed by Jayaraman [9]. The same theory was also applied by Mow et al. [10], Holmes and Mow [11] for the study of articular cartilages. Much of this analysis, has been on one dimensional or purely radial compression without consideration of the influence of shear stresses on the deformable porous media.

The movement of bio-fluids in a physiological system has to be investigated thoroughly in order to solve diagnostic problems that arise in a living body. Some of the bio-fluids like blood are observed to behave like non-Newtonian fluids. Since there is no universal model to describe all non-Newtonian fluids in physiological systems, several models are proposed to explain the behavior of these bio-fluids.

Hayat and Ali [12] investigated the peristaltic motion of a Jeffrey fluid under the effect of a magnetic field. Elshehaway [13] has studied peristaltic transport in an asymmetric channel through a porous medium. Influence of partial slip on the peristaltic flow in a porous medium and a mathematical model of peristalsis in tubes through a porous medium is investigated by Hayat et al. [14]. Vajravelu et al. [15] investigated the peristaltic flow and heat transfer in a vertical porous annulus with long wavelength approximation. Kothandapani and Srinivas [16] made a study on the peristaltic transport of a Jeffrey fluid under the effect of magnetic field in an asymmetric channel. Hayat and Ali [17] investigated the peristaltic motion of a Jeffrey fluid under the effect of a magnetic field.

Among several non-Newtonian models proposed for physiological fluids, Jeffrey model is significant because Newtonian fluid model can be deduced from this as a special case by taking $\lambda_1 = 0$. Further it is speculated that the physiological fluids such as blood exhibit Newtonian and non-Newtonian behaviors during circulation in a living body. Vajravelu et al [18] studied the influence of heat transfer on peristaltic transport of a Jeffrey fluid in a vertical porous stratum. Krishna Kumari et al [19] studied the effect of magnetic field on the peristaltic pumping of a Jeffrey fluid in an inclined channel.

ISSN: 2277-9655
Impact Factor: 1.852

Motivated by these studies, the steady flow of a Jeffrey fluid between deformable porous layers is investigated. The fluid velocity in the free and porous regions is obtained. The expression for the displacement in the porous layer is also obtained. The effects of various physical parameters ϕ^f, G, λ_1, η and d, on the velocity and displacement are discussed through graphs.

Formulation of the Problem

The geometry consists of a steady, fully developed Jeffrey fluid flow through a symmetrical channel with solid walls at $y = \pm h$ and a porous layer of thickness L attached to both walls as shown in Fig.1. By symmetry only half of the channel $y \in [0, \mathrm{h}]$ is considered. The flow region between the plates is divided into two layers. The flow region between the lower plate $y = 0$ and the interface $y = h - L$ is termed as free flow region whereas the flow region between the interface $y = h - L$ and the upper plate $y = h$ is designated as deformable porous region. The fluid velocities in the free flow and deformable are q and v. The displacement in the deformable porous region. The fluid velocity in the free flow region and porous flow region are assumed to be (q, 0, 0) and (v,0,0) respectively. The displacement due to the deformation of the solid matix is taken as (u,0,0). A pressure gradient $\frac{\partial p}{\partial x} = G$ is applied, producing an axially directed flow. Due to the assumption of an infinite channel, there is no x dependence in any of the terms except the pressure.

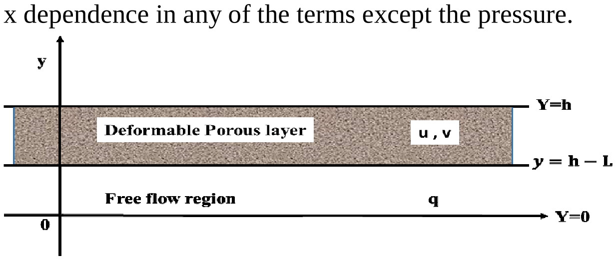

Fig. 1 Physical Model.

With the assumptions mentioned above, the equations of motion in the free flow and deformable porous regions are (following Barry [20])

$$\mu \frac{\partial^2 u}{\partial y^2} = \phi^s G - K v, \tag{1}$$

$$\frac{2\mu_a}{1+\lambda_1} \frac{\partial^2 v}{\partial y^2} = \phi^f G + K v \tag{2}$$

$$\frac{\mu_f}{1+\lambda_1} \frac{\partial^2 q}{\partial y^2} = \frac{\partial p}{\partial x} \tag{3}$$

where μ_a is the apparent viscosity of the fluid in the porous material, λ_1 is Jeffrey parameter, G is the pressure gradient and ϕ^f is the viscous parameter . We note that dot denotes differentiation with respect to time.

Non-Dimensionalization of the Flow Quantities

It is convenient to introduce the following non-dimensional quantities.

$$y = h\hat{y}, \quad u_1 = -\frac{h^2 G_0}{\mu_f}\hat{u}_1, \; v_1 = -\frac{h^2 G_0}{\mu_f}\hat{v}_1, \quad q_1 = -\frac{h^2 G_0}{\mu_f}\hat{q}_1, \; \varepsilon = \frac{L}{h},$$

where G_0 is a typical pressure gradient. In view of the above dimensionless quantities, the equations (1) – (3) take the following form. The hats $(\wedge)$ are neglected here after.

$$\frac{d^2 u}{dy^2} = \phi^s G - \delta v \tag{4}$$

$$\frac{d^2 v}{dy^2} + \delta\eta(1+\lambda_1)v = \phi^f G\eta(1+\lambda_1) \tag{5}$$

$$\frac{d^2 q}{dy^2} = -G(1+\lambda_1) \tag{6}$$

where

$$\delta = \frac{Kh^2}{\mu_f}, \quad \hat{G} = \frac{G}{G_0}, \quad \eta = \frac{\mu_f}{2\mu_a}, \quad G = \frac{dp}{dx}.$$

The parameter δ is a measure of the viscous drag of the outside fluid relative to drag in the porous medium. The parameter η is the ratio of the bulk fluid viscosity to the apparent fluid viscosity in the porous layer.

The boundary conditions are

$$at \quad y = 1: \quad u = v = 0 \tag{7a}$$

$$at \quad y = 0: \quad \frac{dq}{dy} = 0 \tag{7b}$$

ISSN: 2277-9655
Impact Factor: 1.852

$$at \quad y = 1-\varepsilon: \qquad q = \phi^f v$$

(7c)

$$\frac{dq}{dy} = \frac{1}{\eta\phi^f}\frac{dv}{dy}$$

$$= \frac{1}{\phi^s}\frac{du}{dy}$$

(7d)

The first two equations represent the no slip condition for flow at the solid boundary and the symmetry condition along the centre of the channel. Equation (7c) equates the fluid velocity at the interface with the volume averaged velocity of the porous layer. The final two equations arise from the conservation of axial momentum across the fluid-porous layer interface and the assumption that the proportion of the total stress in the porous layer borne by each component is proportional to its volume fraction

Solution of the Problem

Equations (4) to (6) are coupled differential equations that can be solved by using the boundary conditions (7). The displacement and velocities in free flow region and porous regions are obtained as

$$u = c_3 y + c_4 - c_1\delta\frac{y^2}{2} - c_2\frac{\delta}{m^2}e^{(-my)} - \phi^f G\frac{y^3}{6} + \phi^s G\frac{y^2}{2}$$

(8)

$$v = c_1 + c_2 e^{(-my)} + \frac{\phi^f G}{\delta}y \qquad (9)$$

$$q = c_5 y + c_6 - G(1+\lambda_1)\frac{y^2}{2} \qquad (10)$$

where

$$m = \delta\eta(1+\lambda_1), \quad m_1 = \phi^f G\eta(1+\lambda_1), \quad m_2 = \frac{m_1}{m}, \quad a_1 = \frac{\delta}{2}, \quad a_2 = \frac{\delta}{m^2}e^{(-m)}, \quad a_3 = \frac{\phi^s G}{2} - \frac{\phi^f G}{6},$$

$$a_4 = e^{(-m)}, \quad a_5 = -\frac{\phi^f G}{\delta}, \quad a_6 = \phi^f, \quad a_7 = a_6 e^{-m(1-\varepsilon)}, \quad a_8 = -\frac{G(1+\lambda_1)(1-\varepsilon)^2}{2} - \frac{G(\phi^f)^2(1-\varepsilon)}{\delta},$$

$$a_9 = \frac{\phi^f G}{\delta} + G(1+\lambda_1)(1-\varepsilon)\phi^f\eta, \quad a_{10} = -\delta(1-\varepsilon), \quad a_{11} = \frac{\delta e^{-m(1-\varepsilon)}}{m}, \quad a_{13} = a_5 - a_9 a_4, \quad c_1 = a_{13},$$

$$a_{12} = -\phi^s G(1-\varepsilon) + \frac{G(1-\varepsilon)^2}{2} - a_{11}a_9, \quad a_{14} = a_{13}a_6 + a_9 a_7 - a_8, \quad a_{15} = a_{12} - a_{10}a_{13},$$

$$a_{16} = a_1 a_{13} - a_{15} - a_3 + a_2 a_9, \quad c_6 = a_{14}, \quad c_3 = a_{15}, \quad c_4 = a_{16}.$$

Results and Discussions

In this paper, the steady flow of a Jeffrey fluid between thin, deformable porous layers is investigated. When the Jeffrey parameter $\lambda_1 \to 0$, the results (8) to (10) reduce to the corresponding displacement and fluid velocity of Barry et al. (1991). The solutions for the fluid velocity and displacement are evaluated numerically for different values of physical parameters such as Jeffrey parameter λ_1, the pressure gradient G, the viscous parameter ϕ^f, the viscous drag parameter δ and the viscosity parameter η.

The variation of velocity profile of free flow region q and deformable porous layer v with y is calculated, from equations (8) to (10), for different values of ϕ^f and is shown in Fig.2. for fixed $G=1,\ \lambda_1=0.5,\ \delta=2,\ \varepsilon=0.2$ and $\eta=0.5$. We observe that the velocities q increase with the increase in ϕ^f.

The variation of velocity profile of free flow region q and deformable porous layer v with y is calculated, from equations (8) to (10), for different values of pressure gradient G and is shown in Fig. 3,for fixed $\phi^f=0.5,\ \lambda_1=0.5,\ \delta=2,\ \varepsilon=0.2$ and $\eta=0.5$. Here we observe that the velocities q and v increase with the increase in pressure gradient G.

The variation of velocity profile of free flow region q and deformable porous layer v with y is calculated, from equations (8) to (10), for different values of Jeffrey parameter λ_1 and is shown in Fig. 4, for fixed $G=1,\ \phi^f=0.5,\ \delta=2,\ \varepsilon=0.2$ and $\eta=0.5$. Here we observe that the velocities q and v increase with the increase in Jeffrey parameter λ_1.

The variation of velocity profile of free flow region q and deformable porous layer v with y is calculated, from equations (8) to (10), for different values of ratio of bulk fluid viscosity parameter η and is shown in Fig. 5, for fixed $\lambda_1=0.5,\ G=1,\ \phi^f=0.5,\ \delta=2,\ \varepsilon=0.2$. It is found that the velocity q and v increase with the increase in the of ratio of bulk fluid viscosity parameter η.

The variation of displacement u with y is calculated, from equations (9) to (11), for different values of Jeffrey parameter λ_1 and is shown in Fig. 6,for fixed $G=1,\ \delta=2,\ \phi^f=0.5,\ \eta=0.5\ \varepsilon=0.2$. Here we observe that the displacement increases with the increase in Jeffrey parameter λ_1.

The variation of displacement u with y is calculated, from equations (8) to (10), for different values of pressure gradient G and is shown in Fig. 7, for fixed $\lambda_1=0.5,\ \delta=2,\ \phi^f=0.5,\ \eta=0.5\ \varepsilon=0.2$.

Here we observe that the displacement increases with the increase in pressure gradient G.

The variation of displacement u with y is calculated, from equations (8) to (10), for different values of viscous drag parameter δ and is shown in Fig. 8, for fixed $\lambda_1 = 0.5,\ G = 1,\ \phi^f = 0.5,\ \eta = 0.5\ \varepsilon = 0.2$. Here we observe that the displacement increases with the increase viscous drag parameter δ.

The variation of displacement u with y is calculated, from equations (8) to (10), for different values of ratio of bulk fluid viscosity parameter η and is shown in Fig. 9, for fixed $\lambda_1 = 0.5,\ G = 1,\ \phi^f = 0.5,\ \delta = 2,\ \varepsilon = 0.2$. Here we observe that the displacement increases with the increase in of ratio of bulk fluid viscosity parameter η.

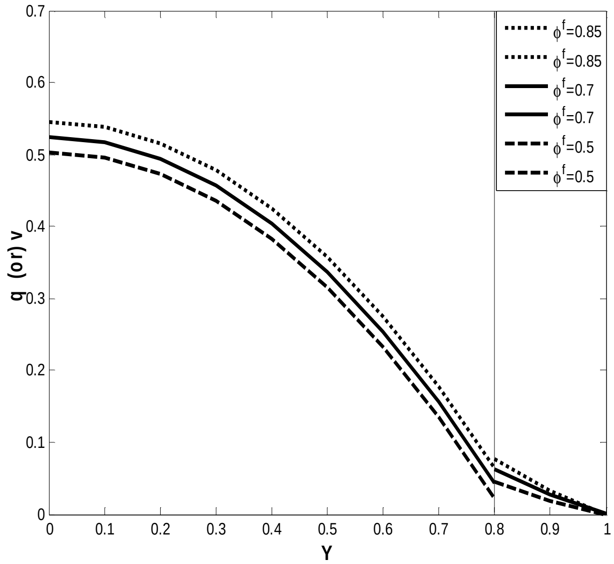

Fig 2. Velocity profile of free flow region q (y=0-0.8) and deformable porous layer v(y=0.8-1) for different values of ϕ^f.

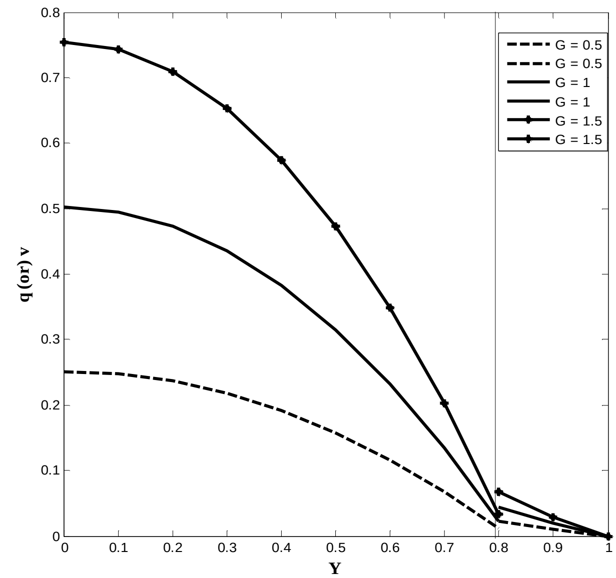

Fig 3. Velocity profile of free flow region q (y=0-0.8) and deformable porous layerv(y=0.8-1) for different values of G.

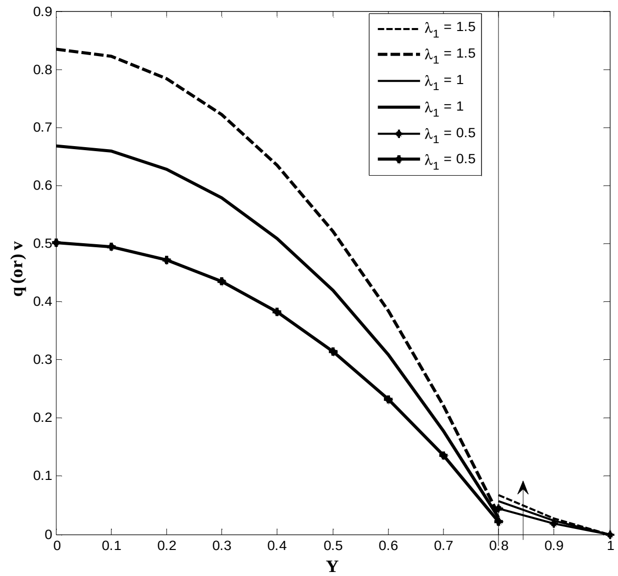

Fig 4. Velocity profile of free flow region q (y=0-0.8) and deformable porous layer v(y=0.8-1) for different values of λ_1.

ISSN: 2277-9655
Impact Factor: 1.852

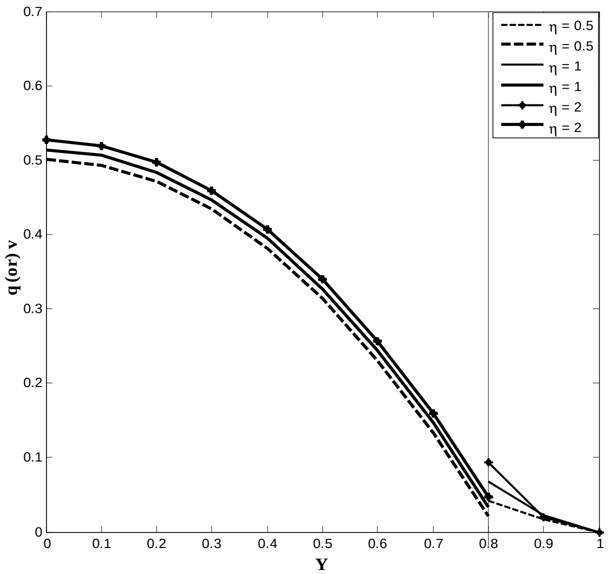

Fig 5. Velocity profile of free flow region q (y=0-0.8) and deformable porous layer v(y=0.8-1) for different values of η.

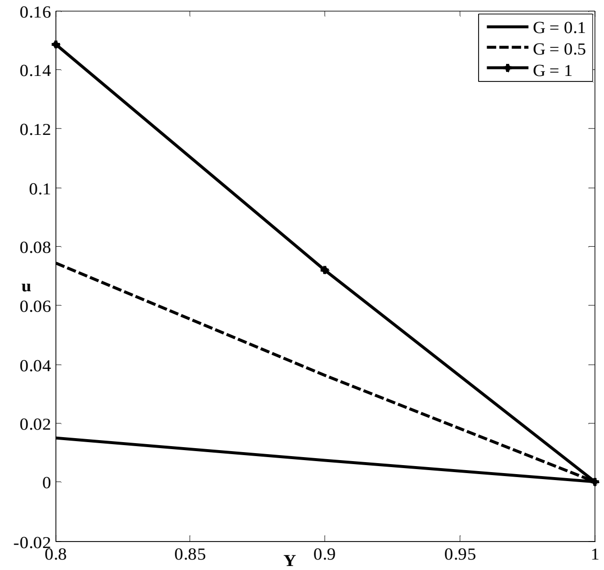

Fig 7. Displacement profile in the deformable porous layer for different values of G.

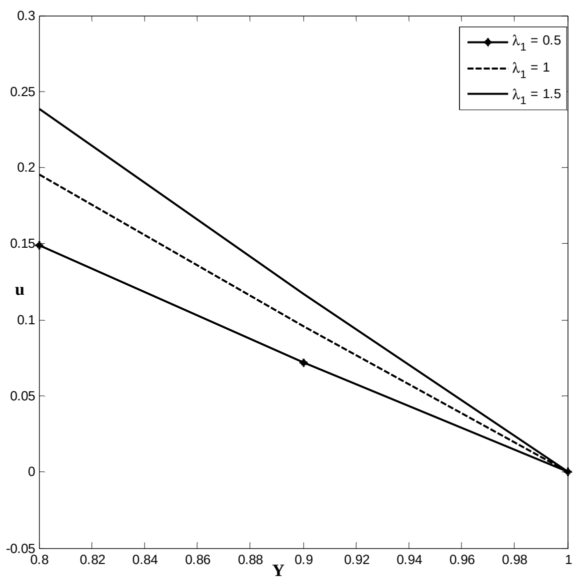

Fig 6.Displacement profile in the deformable porous layer for different values of λ_1.

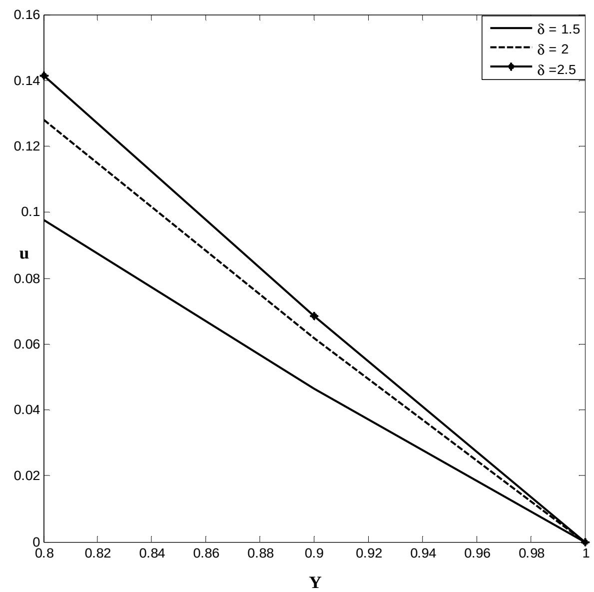

Fig 8. Displacement profile in the deformable porous layer for different values of δ.

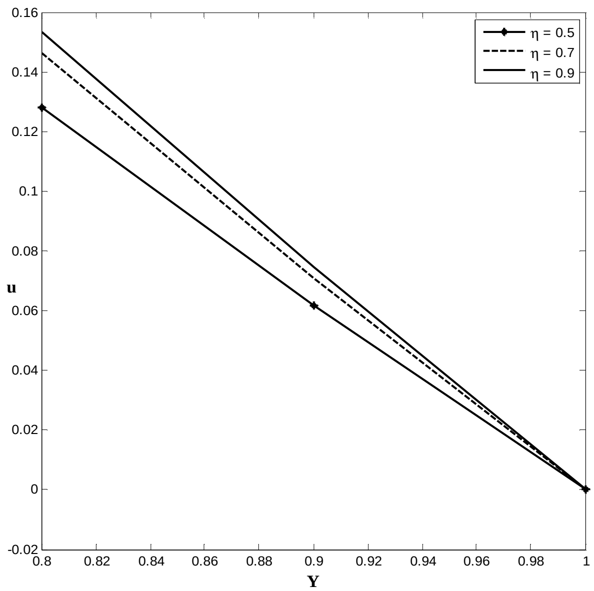

Fig 9. Displacement profile in the deformable porous layer for different values of η.

Acknowledgement

One of the authors Prof. S.Sreenadh expresses thanks to UGC for providing financial support through the Major Research Project to undertake this work.

References

[1] *Terzaghi, K., Erdbaumechanik auf BodenphysikalischenGrundlagen. Deuticke (1925).*

[2] *Biot, M.A., General theory of three-dimensional consolidation. J. Appl. Phys.,* ***12*** *(1941),155-164.*

[3] *Biot, M.A., Theory of elasticity and consolidation for porous anisotropic solid. J. Appl. Phys.,* ***26****(1955), 82-185.*

[4] *Biot, M.A., Mechanics of deformation and acoustic propagation in porous media. J. Appl. Phys.,* ***27*** *(1956).*

[5] *Atkin, R.J. and Craine, R.E.,Continuum theories of mixtures: Basic theory and historical development. Quart. J. Appl. Math.,* ***29*** *(1976), 209-244.*

[6] *Bowen, R.M., Incompressible porous media models by the theory of mixtures. Int. J. Engng. Sci.,* ***18****(1980), 1129-1148.*

[7] *Bedford, A. and Drumheller, D. S., Recent advances, theory of immiscible and structured mixtures. Int. J. Engng. Sci.,* ***21*** *(1983), 863-960.*

[8] *Mow, M.H. Holmes and Lai, M., fluid transport and mechanical properties of articular cartilage: a review. J. Biomechanics,* ***17*** *(1984) 377-394*

[9] *Jayaraman, G., Water transport in the arterial wall- A theoretical study. J. Biomechanics,* ***16*** *(1983), 833- 840*

[10] *Mow,V.C., Kwan, M.K.,Lai,W.M. and Holmes,M.H., A finite deformation theory for non linearly permeable soft hydrated biological tissues. Frontiers in Biomechanics, G.Schmid-Schoenbein, S. L.Y. Woo and B. w. zweifach (eds), Springer-Verlag, New York, (1985) 153-179.*

[11] *Holmes,M.H. and Mow,V.C., The nonlinear characteristics of soft gels and hydrated connective tissues in ultrafiltration. J. Biomechanics,* ***23****, (1990), 1145-1156.*

[12] *Hayat, T and Ali, N., Physica A Statistical Mechanics and its Applications,* ***370*** *(2006), 225- 239.*

[13] *Elshehaway, E.F., Eldabe, N.T., Elghazy, E.M. and Ebaid, A., Peristaltic transport in an asymmetric channel through a porous medium", Appl. Math. Comput.,* ***182,*** *(2006), 140–150.*

[14] *Hayat,T., Khan, S.B. and Khan , M., The influence of Hall current on the rotating Oscillating flows of an Oldroyd-B fluid in a porous medium. Nonlinear Dyn.,****47,*** *(2007), 353- 362*

[15] *Vajravelu K, Radhakrishnamacharya, G and Radhakrishnamurty V., Peristaltic flow and heat transfer in a vertical porous annulus, with long wavelength approximation. Int J Non-linearMech,* ***42*** *(2007),754–9.*

[16] *Kothandapani, M. andSrinivas ,S., On the influence of wall properties in the MHD peristaltic transport with heat transfer and porous medium, Phys A.* ***372*** *(2008) 4586-4591.*

[17] *Hayat T, Ali N. Peristaltic motion of a Jeffrey fluid under the effect of a magnetic field in a tube. Commun Non-linear Sci Numer Simul,* ***13*** *(2008), 1343–52.*

[18] *K. Vajravelu, S. Sreenadh and P. Lakshminarayana. The influence of heat transfer on peristaltic transport of a Jeffrey fluid in a vertical porous stratum. CommunNonlinear Sci NumerSimulat,****16*** *(2011) 3107–3125*

[19] *Krishna Kumari,S.V.H.N., Ramana Murthy,M.V., Ravi Kumar, Y.V.K. andSreenadh,S., Peristaltic pumping of a Jeffrey fluid under the effect of a magnetic field in an inclined channel, Appl.Math. Sciences,* ***5****(2011),447 – 458.*

ISSN: 2277-9655
Impact Factor: 1.852

[20] *Barry, S.I., Parker, K.H. and Aldis, G.K. Fluid flow over a thin deformable porous layer. Journal of Applied Mathematics and Physics (ZAMP), 42, (1991), 633-648.*